COMP 3200
Artificial Intelligence

Lecture 11
Voting Algorithms

Lecture Resources

- Veritasium, YouTube
  - Why democracy is Mathematically Impossible
- CGP Grey Voting Series, YouTube
  - Why Your Vote Doesn't Count
  - The Mathematically Superior Voting System
  - Full Playlist
- Other Sources
  - Voting Theory: Plurality Method and Condorcet Criterion

Voting

- Voting is a method for aggregating individual preferences for selection into a collective decision
- Used in situations where a single shared outcome must be chosen (ex: choose a leader), but agreement through other means is infeasible
- Individuals express preferences over a shared set of options, and then a predefined set of rules determines how to combine those into an outcome
- Algorithms used factor heavily into outcome

Voting in Society

- Voting used widely in political systems to choose representatives, pass laws, etc
- Outside politics, used to select committees, hire employees, grants, awards, etc
- Real-world voting systems constrained by need for simplicity and public acceptance
  - Can conflict with optimal decision making

Voting in Computer Science

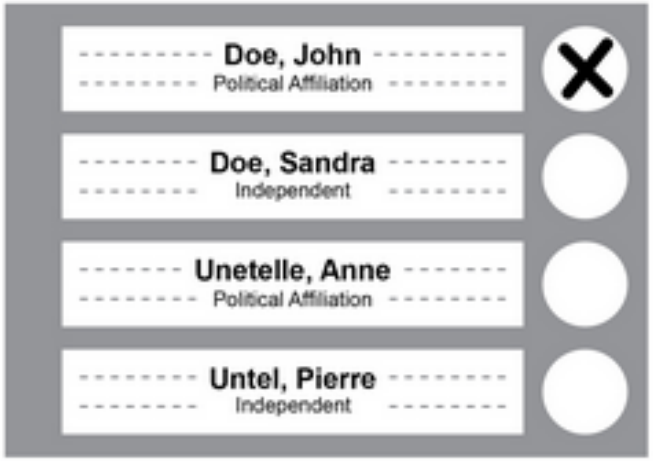
- Voting can arise in situations where multiple agents, processes, or users must coordinate on a shared outcome
- Examples: distributed systems, multi-agent decision making, recommender systems
- Computationally, voting is just a preference aggregation algorithm

Voting Algorithm

- Voting can be viewed as a function
- Inputs
  - Set of candidates to select from
  - Individual preferences / ballots
- Outputs
  - Single winner, set of winners (or)
  - Ranking of candidates

Voting Inputs (ballots)

- Ballots can vary dramatically based on the voting algorithm and desired outcome
- Examples:
  - Single choice of candidate (select one)
  - Approval of candidates (select many)
  - Ranking of candidates (best to worst)
  - Numerical scores (1 – 10)



Vote for any number of options.

- [ ] Joe Smith
- [x] John Citizen
- [ ] Jane Doe
- [ ] Fred Rubble
- [x] Mary Hill

Sample Ballot

Fill in the ovals by ranking candidates in order of preference	1st Choice	2nd Choice	3rd Choice
Candidate A	1	**2**	3
Candidate B	**1**	2	3
Candidate C	1	2	**3**

Candidates		Score *each* candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	**0** 1 2 3 4 5 6 7 8 9
2: Candidate B	→	0 1 2 3 4 5 6 7 8 **9**
3: Candidate C	→	0 1 2 3 4 5 6 **7** 8 9

Voting Outcomes / Outputs

- Different elections may have different desired outcomes / decision making
- Outcomes reflect the goal of the election
- Examples
  - Choose a single winner (1 from N)
  - Choose a set of winners (X from N)
  - Produce a ranking (1 to N)

Voting Outcome Examples

- Single winner: a political system may require choosing a single leader
- Set of winners: choosing a town council of 6 people from a set of 19 candidates
- Ranking: a hiring committee is asked to rank candidates from a pool of applicants, offers are sent in that ranked order

Voting System Considerations

- Fairness: should treat voters and candidates symmetrically, avoid biases
- Simplicity: how easy is it to understand
- Expressiveness: how much information voters are allowed to convey about candidates
- Security: resistance to manipulation
- Computability: can we run the algorithm efficiently in (possibly) real-world scenarios

Voting System Goals

- The goal of a voting system is typically to maximize the overall happiness or satisfaction of the results of the process
- If each voter were to assign a numerical utility value to each candidate winning, we want the voting result to maximize the sum of that value over all voters

Voting Algorithm Taxonomy

- Voting systems can be classified by how much information they extract from each voter and how that info is aggregated
- Classification helps compare systems and allow people to understand how they work
- Voting systems differ primarily on how voters assign preferences to candidates

Voting System Types

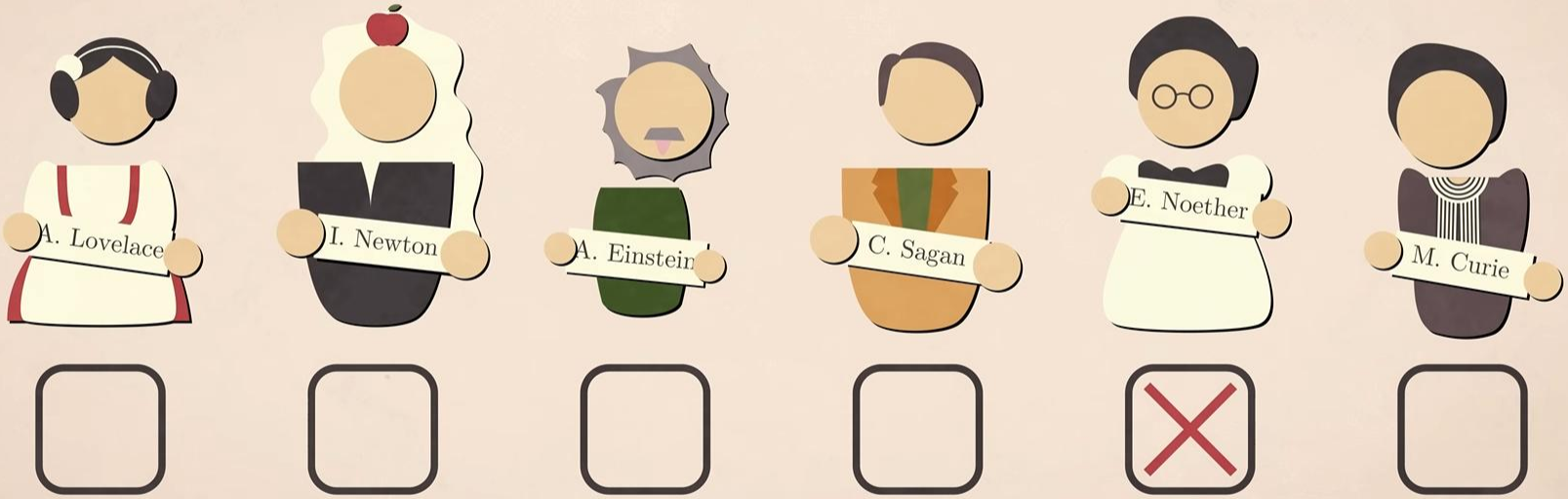
- Single-choice (plurality-based)
  - Restrict voters to exactly one option
  - Simplest ballots, but least voter information
- Ranked (preference ordering)
  - Ballots are total or partial ranking of candidates
  - More complex, but more voter information
- Approval based
  - Voter selects 1 or more candidates they approve of
  - Simple but discards ranking information

Voting System Types

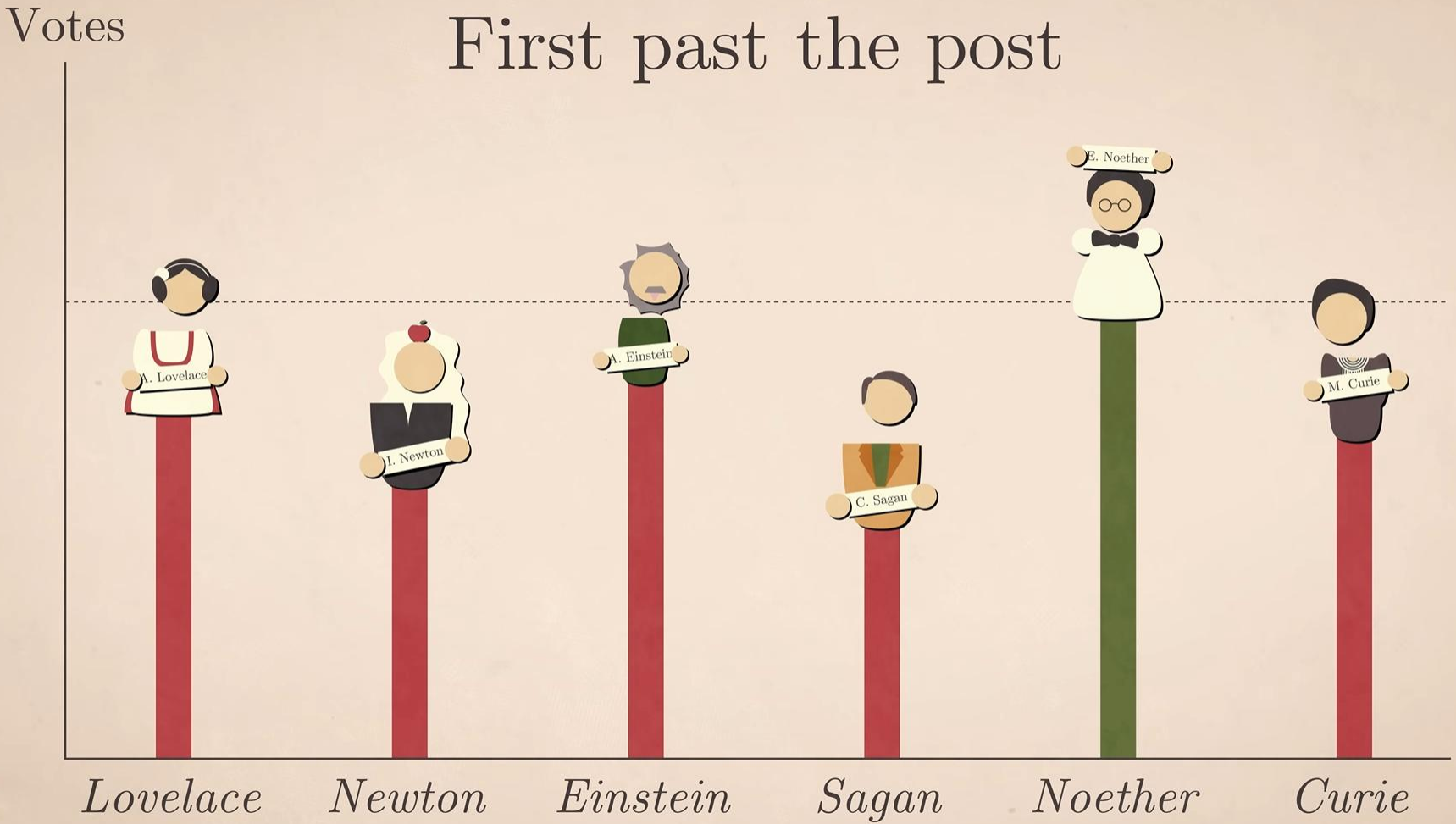
- Score based
  - Voters directly assign numerical scores to candidates
- Positional (rank-score) based
  - Candidates given points based on ballot ranking
- Pairwise / Condorcet
  - Compare candidates head-to-head
  - Select candidate that would win against all others
- Proportional Representation
  - Allocates multiple seats instead of single winner
  - May more accurately reflect voter choices

First-Past-the-Post (FPTP)

- Simplest voting algorithm
  - Ballot = One vote for one candidate
  - Winner = Candidate with most votes
- Plurality-based voting system
  - Winner doesn't need a majority of votes, just highest
- Linear space and time complexity
  - Count each vote once and sum the values
  - Easy to implement for real-life elections


A. Lovelace
I. Newton
A. Einstein
C. Sagan
E. Noether
M. Curie

First past the post




First Past the Post
44 Countries

FPTP Positives

- Simple counting procedure
- Simple candidate selection (one choice)
- Very easy to understand
  - Voters typically trust what they understand
- Efficient to implement and tabulate
  - Simpler vote = harder to make mistakes
  - Easy to count / recount = faster results

FPTP Negatives

- Single choice voting discards information about secondary preferences, relative strength of candidates, acceptable compromises, etc
  - You may be happy with A or B, but can only choose A
- Two voters with different overall preferences may end up casting identical ballots
- Loss of information limits the system's ability to reflect the true nature of the voter preferences

9%

18%

19%

13%

20%

15%

6%

Minority Rule Often Happens in FPTP

FPTP Spoiler Effect

- Close election between candidates A and B
- Candidate C appears that has a close political strategy to candidate A
- Some that vote for C may prefer A to B but cannot express that preference
- Candidate C “takes” enough votes from A that B has most votes and wins election

FPTP Spoiler Example

- Class votes for end of term party food
- Choices: Hamburgers or Pepperoni Pizza
- Class Preference
  - 60% prefer Pepperoni Pizza
  - 40% prefer Hamburgers
- In a head-to-head vote, the class preference is properly represented

Voting for Food - FPTP

40%

60%

Pizza Wins

FPTP Spoiler Example

- What if there was a 3rd candidate for the choice of food: Hawaiian Pizza
- Voting changes for Pepperoni voters:
  - Some may prefer and switch to Hawaiian
  - Some won’t switch but they would still like to have Pizza over a Hamburger
- Effect: Pizza vote now gets “split”

Voting for Food - FPTP

40%

60%

Voting for Food – FPTP Spoiler

40%

35%

25%

Voting for Food – FPTP Spoiler

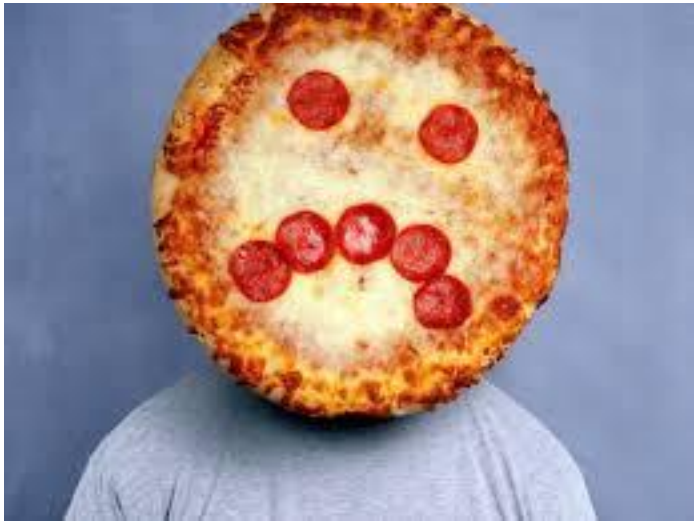
40%

35%

25%

FPTP Spoiler Example

- First Example:
  - 60% vote Pepperoni Pizza
  - 40% vote Hamburgers
  - Pizza Wins
- Second Example
  - 25% vote Hawaiian pizza
  - 35% vote Pepperoni pizza
  - 40% vote Hamburgers
  - Hamburgers wins
- Pizza voters had no way to express secondary preference



Political Note

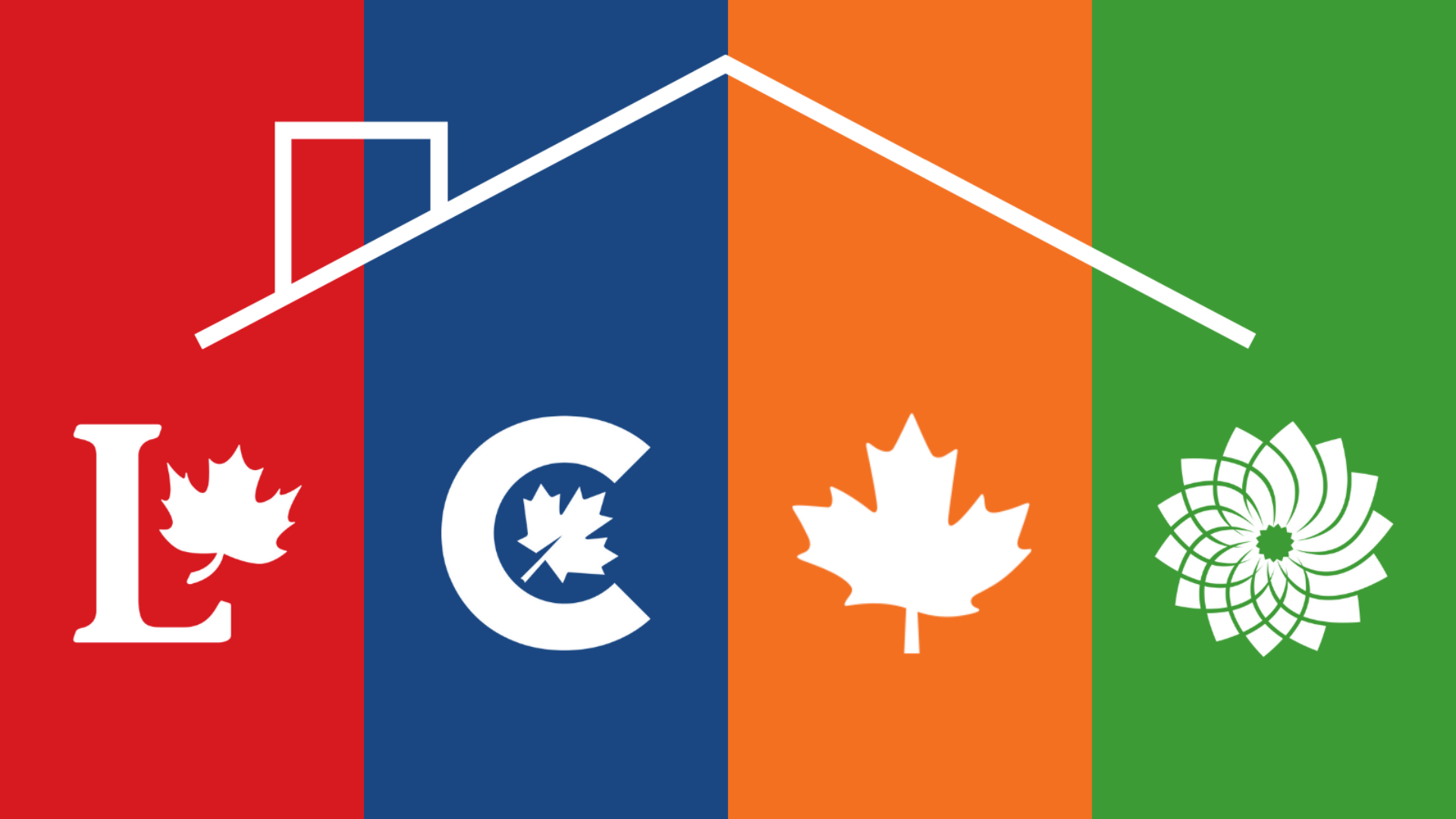
- With following examples, my goal is to show you the real-world effects that can happen because of FPTP voting
- I am not trying to comment on the political parties, their policies, or any individual candidates, just to show the mathematical outcomes of FPTP voting systems
- Whether a spoiler effect gives an advantage one party or another party doesn't matter for algorithmic scrutiny
- Spoiler effects are bad for everyone because our goal is to maximize overall happiness for everyone who is voting
- The goal here is to discuss the math, not the politics

Real World Spoiler Effect

2000 United States presidential election in Florida[11][12]					
Party	Candidate	Running mate	Votes	Percentage	Electoral votes
Republican	**George W. Bush**	**Dick Cheney**	**2,912,790**	**48.847%**	**25**
Democratic	Al Gore	Joe Lieberman	2,912,253	48.838%	0
Green	Ralph Nader	Winona LaDuke	97,488	1.64%	0
Reform	Patrick Buchanan	Ezola Foster	17,484	0.29%	0
Libertarian	Harry Browne	Art Olivier	16,415	0.28%	0
Natural Law	John Hagelin	Nat Goldhaber	2,281	0.04%	0
Workers World	Monica Moorehead	Gloria La Riva	1,804	0.03%	0
Constitution	Howard Phillips	Curtis Frazier	1,371	0.02%	0
Socialist	David McReynolds	Mary Cal Hollis	622	0.01%	0
Socialist Workers	James Harris	Margaret Trowe	562	0.01%	0
		Write-in	36	<0.01%	—
		Totals	**5,963,110**	**100.00%**	**25**

Florida 2000 Election

- Republican George Bush narrowly beat Democrat Al Gore in Florida election
- Ralph Nader got far more votes than the difference between Bush and Gore
- Many Nader voters probably would have preferred Gore to Bush but couldn’t express
- With Nader Bush wins, without Gore wins



Canadian Elections

- A similar spoiler effect happens quite often in Canadian elections
- 3 main parties (sometimes more)
  - Conservative, Liberal, NDP
  - Liberal and NDP are more closely politically aligned than they are with Conservative
  - Voters cannot express preferences
- Similar to Pizza / Hamburger example

2025 NL Provincial Election

Party	Total Votes	Seats won
PC	88,511	21
Liberal	86,668	15
NDP	16,850	2

PC party votes carried 44% more power than Liberal and NDP votes in the final election results

Two Party Convergence

- Spoiler effect happens because voters in a FPTP system cannot express any information in ballot other than their first choice
- Eventually this effect forces all FPTP voting systems to converge toward a two-party system, since more politically aligned parties will only serve to split the vote further (Duverger's Law)
- In an ideal world, more political parties would increase overall voter choice and happiness

Strategic Voting

- Another byproduct of FPTP is that voters will strategically vote for a party they think can win, rather than their overall true preference
- Seen historically in voting movements such as "ABX" (Anything But X) where some voters will switch votes just so party X does not win
- In an ideal voting system, candidates could vote for their true preference without any need to consider strategic voting

Runoff Elections

Runoff Elections

- One attempt to fix FPTP is via Runoffs
- Instead of mere plurality, require majority
- Runoff Algorithm
  1. Hold a normal FPTP election
  2. If someone gets >= 50% votes, they win
  3. If not, eliminate candidate with the fewest votes and hold another election without them

Voting for Food – Runoff

40%

35%

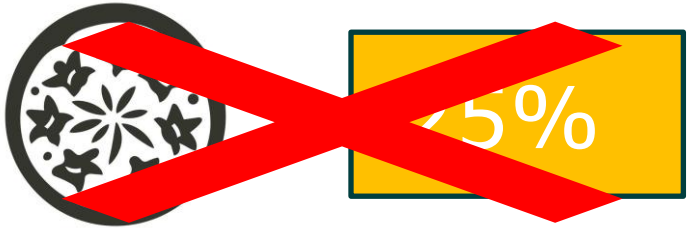
25%

No Candidate
>= 50%
Eliminate Lowest
Vote Again

Voting for Food – Runoff

40%

35%



25%

Voting for Food – Runoff

Pepperoni Pizza Wins
>= 50% Votes

Runoff Election Motivation

- Addresses the problem of FPTP where a winner could be opposed by majority
- Central idea is that the eventual winner will have broader support than any single plurality FPTP would have
- Reduced spoiler effect by allowing voters to vote their preferred candidate knowing another vote may come later

Runoff Election Negatives

- Getting people to show up to vote once is hard enough, much harder for multiple rounds of elections on the same issue
- Increased complexity of election rules and need to organize and run more votes
- Time complexity goes from linear to possibly quadratic in the worst case

What if we didn’t need to have multiple elections?

Ranked Choice Voting

Instant Runoff Voting (IRV)

- Also called Ranked Choice Voting (RCV)
- Vote ballots rank all candidates from 1-N based on their preference. Non-ranked candidates will be ignored for voting.
- Instant runoff means that we can hold a separate runoff vote instantly by using candidates next choice. One election only

IRV / RCV Note

- IRV is a specific voting algorithm where candidates are eliminated one at a time based on first-choice votes
- RCV is a broader term for voting systems in which voters rank candidates on ballots
- They are not exactly the same, but names are often used interchangeably

IRV / RCV Algorithm

1. Voters cast ballots ranking all candidates
2. Count each candidate's #1 rank votes
3. If any candidate has >50% rank 1 votes
    1. Election is over, that candidate wins
4. If nobody has more than 50% rank 1 votes
    1. Remove candidate with fewest R1 votes from election
    2. Votes for eliminated candidate go to next ranked choice
5. Repeat steps 2-4 until winner is found

Voting for Food – IRV / RCV

40%

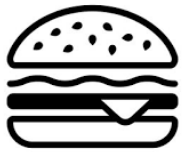
35%

25%

	1st Choice	2nd Choice	3rd Choice
P Pizza	1	**2**	3
H Pizza	**1**	2	3
Burger	1	2	**3**

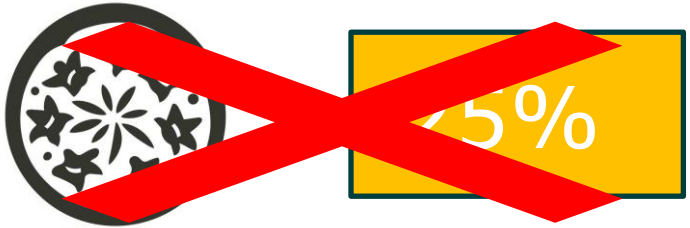
Rank 1 Votes

Voting for Food – IRV / RCV



40%

35%



25%

	1st Choice	2nd Choice	3rd Choice
P Pizza	1	2	3
H Pizza	1	2	3
Burger	1	2	3

H Pizza Eliminated

Voting for Food – IRV / RCV

40%

35%

25%

	1st Choice	2nd Choice	3rd Choice
P Pizza	1	**2**	3
H Pizza	**1**	2	3
Burger	1	2	**3**

Voting for Food – IRV / RCV

40%

35%

25%

	1st Choice	2nd Choice	3rd Choice
P Pizza	1	2	3
H Pizza		2	3
Burger	1	2	3

For voters who chose H Pizza
Vote transferred to next choice

Voting for Food – IRV / RCV

No Need to Hold
Separate runoff election

Problem With RCV

- RCV is far better than FPTP in pratcie
- But it’s possible that doing worse in a first voting round can lead to better performance in the final results
- This is completely dependent on the exact results of ither candidates with similar / conflicting views

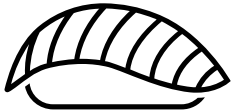
Voting for Food – Vote 1

45%

HATE Sushi

30%

Middle Ground



25%

HATE Burgers

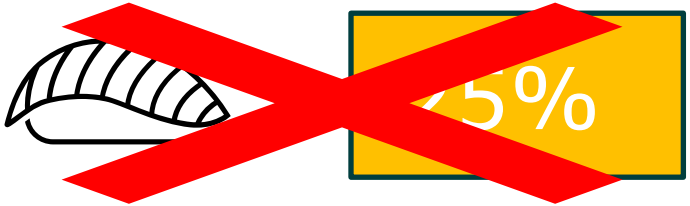
Voting for Food – Vote 1

45%

HATE Sushi

30%

Middle Ground



25%

HATE Burgers

Voting for Food – Vote 1

45%

HATE Sushi

30%

25%

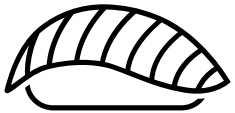
Middle Ground

Sushi Lovers Chose Middle Ground Pizza Option

Voting for Food – Vote 1

45%

30%



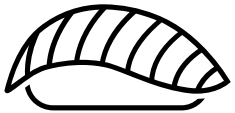
25%

Voting for Food – Vote 2

39%

Something Gross

30%

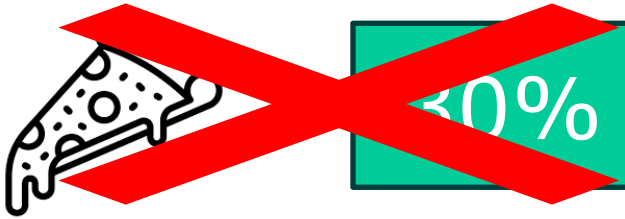


31%

Chose Sushi

Voting for Food – Vote 2

39%



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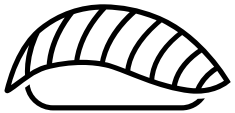
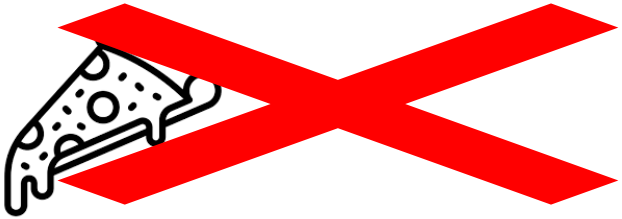
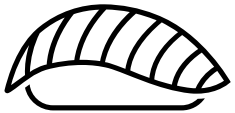


31%

Voting for Food – Vote 2





Doing Worse Initially
Made Burger Win

Positional Voting

Positional Voting

- Another variation of a ranked voting method where candidates are given numerical score based on ballot ranking
- After all ballots have been cast, candidate with the highest score sum wins
- Produce decent results, but are susceptible to manipulation by assigning low scores to candidates you don’t like

Borda Method

- Type of positional voting where N candidates are ranked 1st to last choice
- Scores given as follows:
  - 1st choice = N-1 points
  - 2nd choice = N-2 points
  - …
  - Nth choice = 0 points
- Points summed with highest winning

Rank any number of options in your order of preference.

- [ ] Joe Smith
- [1] John Citizen
- [3] Jane Doe
- [ ] Fred Rubble
- [2] Mary Hill

Ranking	Candidate	Formula	Points
1st	Andrew	$n - 1$	**3**
2nd	Brian	$n - 2$	**2**
3rd	Catherine	$n - 3$	**1**
4th	David	$n - 4$	**0**

Candidate	*U* Points	*V Points*	*W* points	Total
Andrew	3	3	0	6
Brian	2	2	3	7
Catherine	1	1	2	4
David	0	0	1	1

Driver's Points

Mario Kart 64 Points System

If a player comes in fourth place or better, they receive Driver's Point
to the next course. Place fifth or lower and a menu will appear after you reach the goal.
Select RETRY to restart the same course.

1st Place	2nd Place	3rd Place	4th Place
9 Points	6 Points	3 Points	1 Point

4th Place Or Better

5th Place Or Lower


5/6
1 Nathan +27 3563
2 RockSJames +16 6855
3 Sportdrink +9 8166
4 athena +14 4981
5 LordCactus -2 9897
6 sam <3 +12 2964
7 Chuckie F -6 8501
8 ithomZ -7 7563
9 Jendawgg -8 6862
10 Zeeonya +1 1039
11 Nova -1 1085
12 bryce3337 -5 1000

Eurovision Song Contest Voting

Rank	Points
1st	12
2nd	10
3rd	8
4th	7
5th	6
6th	5
7th	4
8th	3
9th	2
10th	1

Arbitrary Scoring Methods

- Many other score-based methods exist
- They are all susceptible to score manipulation in a similar way to Borda
- They also suffer similarly to product rating systems, many ballots end up having only min or max scores allocated
- Will not cover any more of them here

Condorcet Method

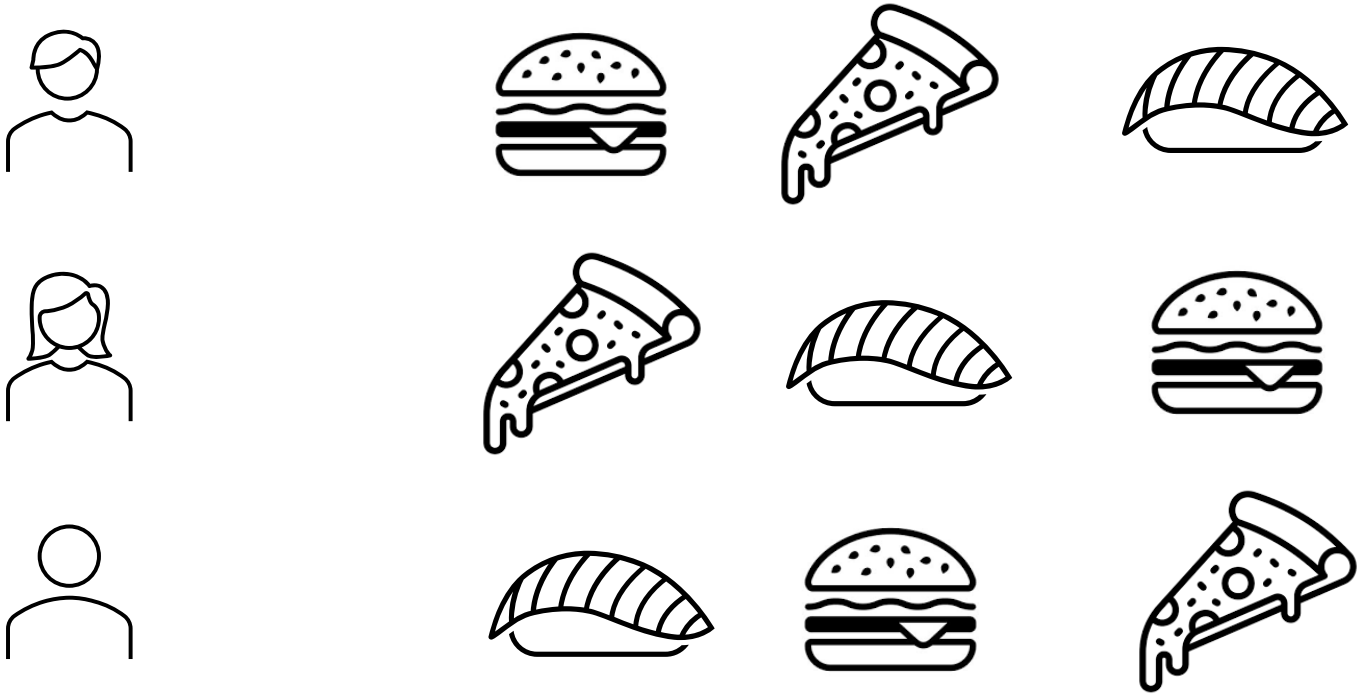
Condorcet Method

- Main idea: if there is to be a single winner, they should beat every other candidate in a head-to-head election
- May seem like it is inefficient since we would need to have many rounds of voting
- However, using ranked choice ballets we can simulate all of those 1v1 elections

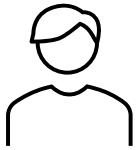
Condorcet Paradox

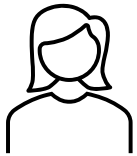
- Condorcet method seems very fair
- However, it can suffer to the Condorcet Paradox, similar to Rock-Paper-Scissors
  - Rock beats Scissors
  - Scissors beats Paper
  - Paper beats Rock
- Possible for election to have no winner

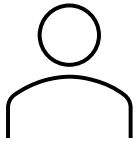
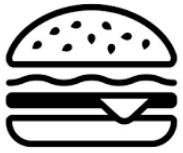
Condorcet Paradox Example



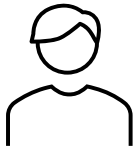
Condorcet Paradox Example

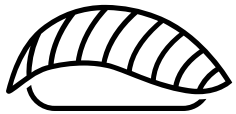
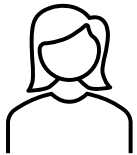


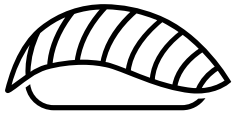
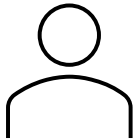
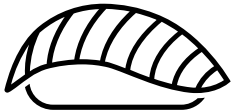



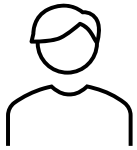

Condorcet Paradox Example

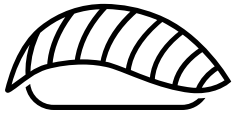
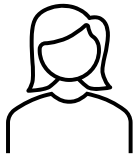
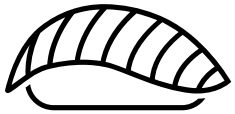


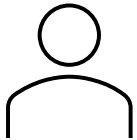
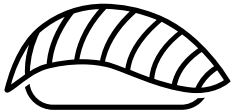



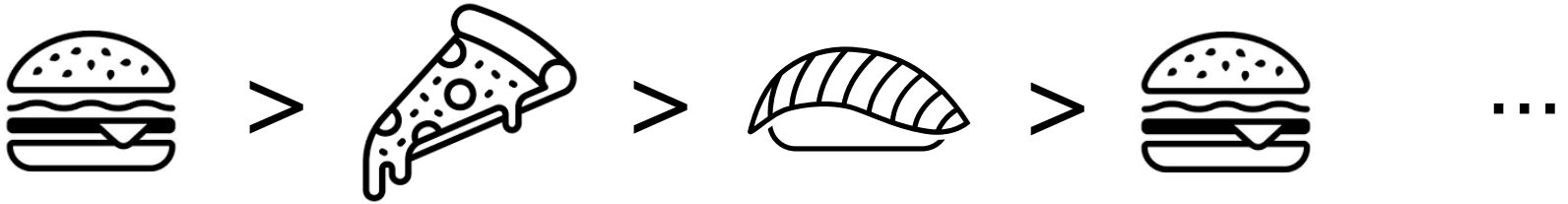





Condorcet Paradox Example



Infinite Preference Loop

Kenneth Arrow's 5 Criteria

- Unanimity: If every individual prefers one choice to another, the final group ranking should also prefer it
- Freedom from Irrelevant Alternatives: If a choice is removed, other choices order doesn't change
- Individual Sovereignty: Each individual should be able to order choices in any way, and indicate ties
- Uniqueness of Group Rank: Method should yield the same result when applied to same ballots every time
- Non-Dictatorship: The preferences of any individual should never outweigh all others in the group

Arrow's Impossibility Theorem

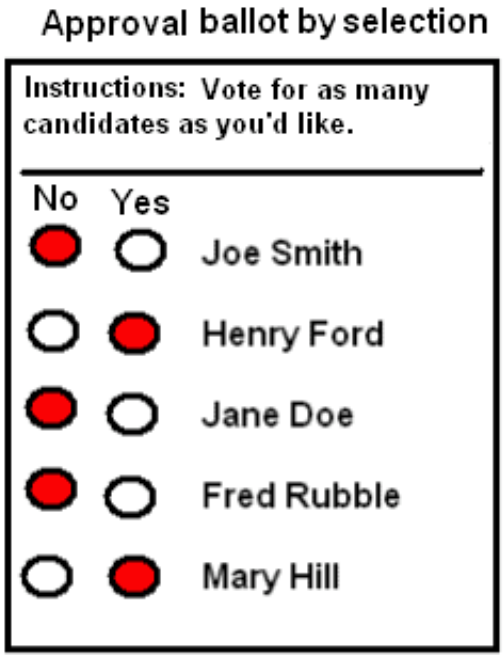
- Arrow described 5 criteria we would ideally want to have in a voting algorithm
- Also showed that for any ranked choice voting system, it is impossible for all 5 criteria to be true at the same time
- We must always give up something
- Won Nobel Prize in Economics

Approval Voting

Approval Voting

- Instead of requiring a ranking of all candidates, just select those you approve of
- "I would be fine with any of these"
- Winner is the candidate that receives the most overall approval votes
- Loses some ranking information, but avoids some pitfalls of ranked choice voting

Approval Voting



Vote for any number of options.

- [ ] Joe Smith
- [x] John Citizen
- [ ] Jane Doe
- [ ] Fred Rubble
- [x] Mary Hill

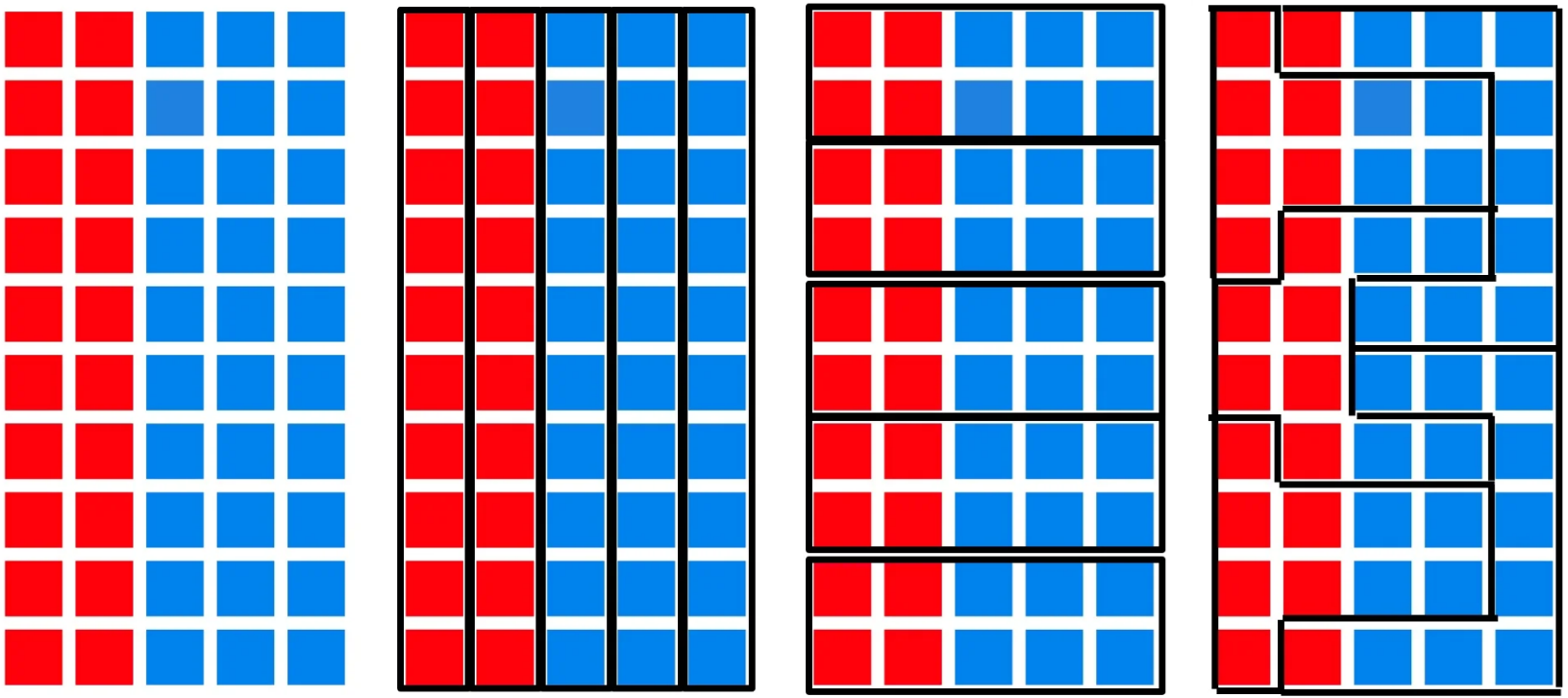
Approval Voting

- Often used in practice for voting when there are multiple winners (committees)
- Town Council: Vote for 6 of 19 candidates for 6-person town council membership
- Can also be used for single winner, but is sometimes confusing to voters
  - “Why vote for multiple if only one wins?”

Other Voting Methods

- There are many other voting algorithms, but most are variations on these types
- There is no single provable best voting method to use for all possible elections
- FPTP is provably terrible in most situations and RCV / Approval do better in practice
- We have only scratched the surface

Gerrymandering



Gerrymandering

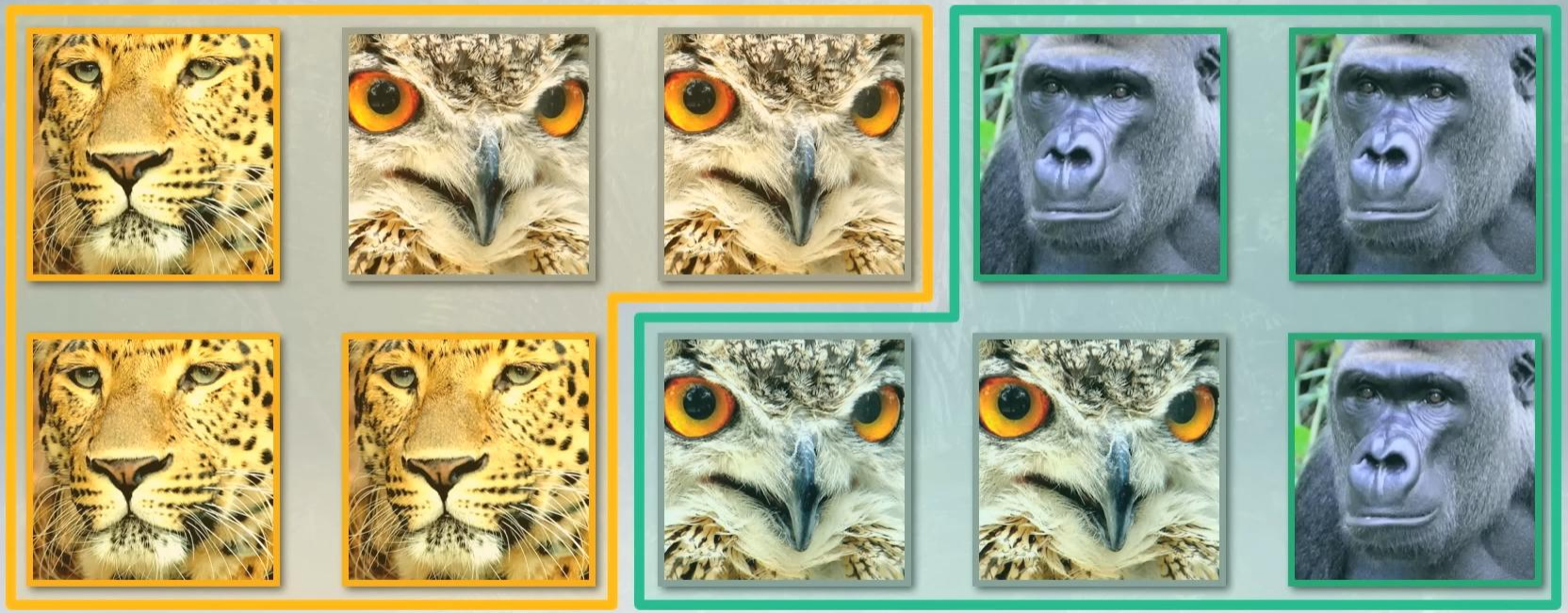
- The manipulation of voter grouping boundaries to give an advantage to one or more groups in an election
- Is often done based on geographical boundaries / other historical reasons
- FPTP voting is particularly vulnerable to this type of manipulation

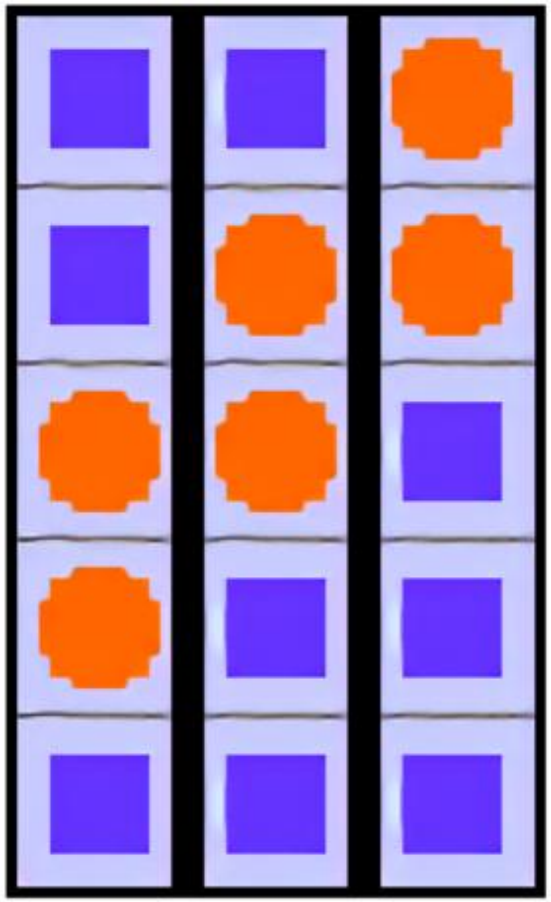




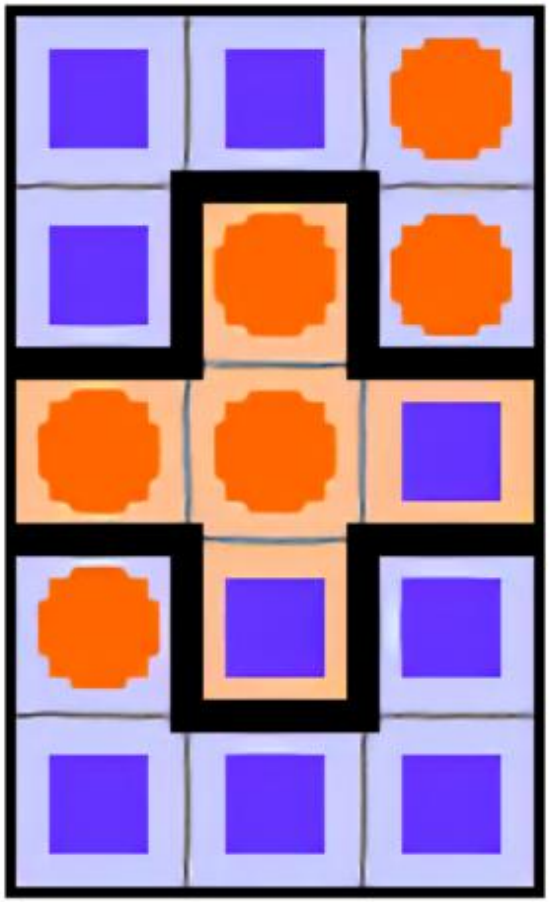




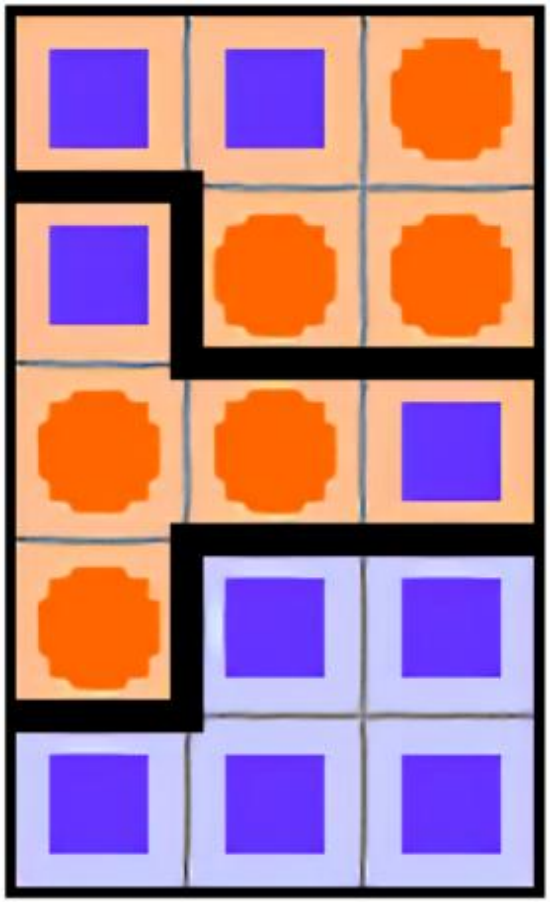





Blue 3-0

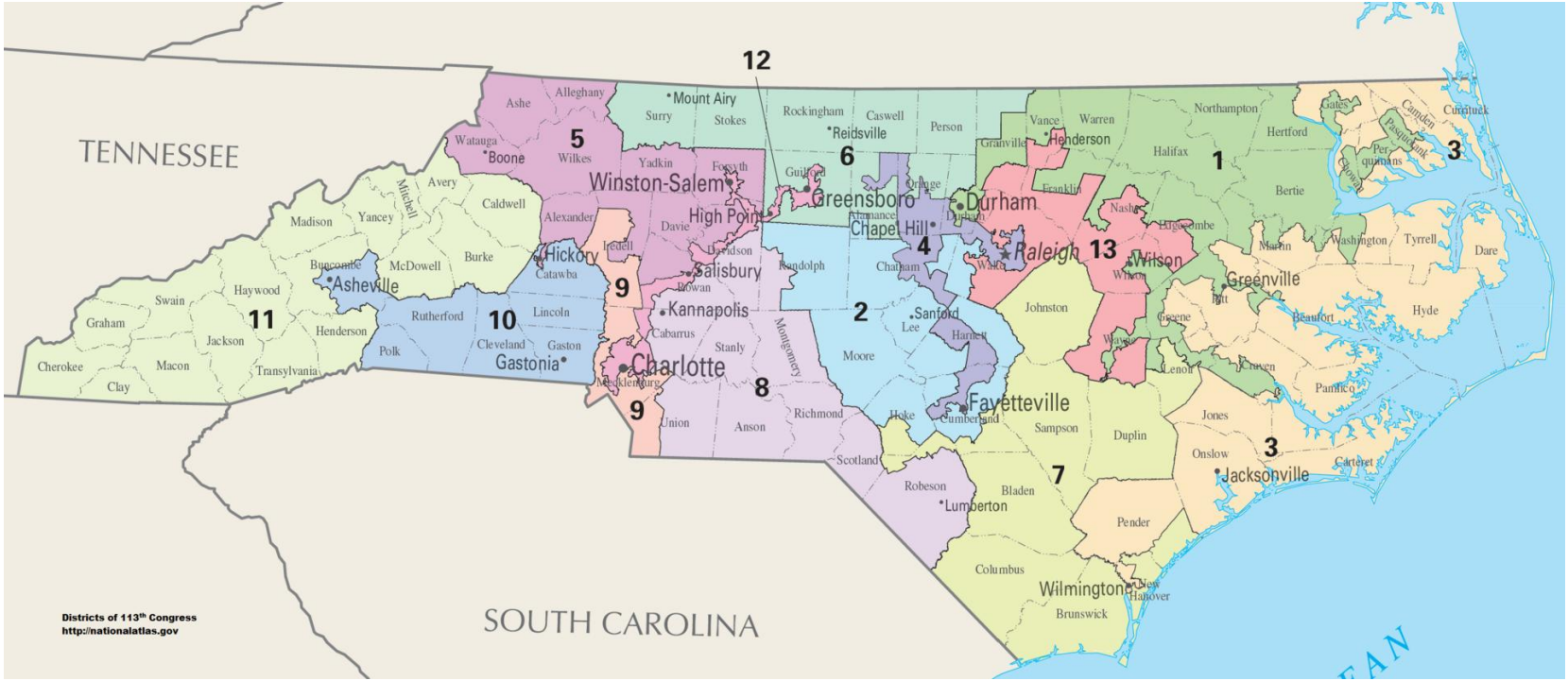


Blue 2-1



Orange 2-1

Real-World Districts



Single Transferrable Vote (STV)

- Ranked-choice voting system where multiple winners are selected instead of 1
- Can help solve issues of representation as well as mitigate gerrymandering
- Algorithm is a little complicated but works very well in practice overall
  - CGPGrey Video on STV (watch this video)

Exam Questions

- Given ballot data, calculate winner for
  - FPTP, IRV / RCV, Borda, Condorcet
- Explain pros / cons of voting methods
- Gerrymander election boundaries so that a specific candidate wins / loses